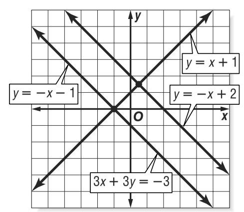
7-1 Study Guide and Intervention

Graphing Systems of Equations

Number of Solutions Two or more linear equations involving the same variables form a **system of equations**. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. The table below summarizes information about systems of linear equations.

Graph of a System	intersecting lines	same line	parallel lines
Number of Solutions	exactly one solution	infinitely many solutions	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent

Example **Use the graph at the right to determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions.**



a. $\boldsymbol{y = -x + 2}$
$\boldsymbol{y = x + 1}$

Since the graphs of $y = -x + 2$ and $y = x + 1$ intersect, there is one solution.

b. $\boldsymbol{y = -x + 2}$
$\boldsymbol{3x + 3y = -3}$

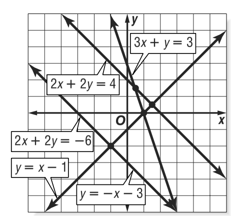
Since the graphs of $y = -x + 2$ and $3x + 3y = -3$ are parallel, there are no solutions.

c. $\boldsymbol{3x + 3y = -3}$
$\boldsymbol{y = -x - 1}$

Since the graphs of $3x + 3y = -3$ and $y = -x - 1$ coincide, there are infinitely many solutions.

Exercises

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



1. $y = -x - 3$
$y = x - 1$
one

2. $2x + 2y = -6$
$y = -x - 3$
infinitely many

3. $y = -x - 3$
$2x + 2y = 4$
none

4. $2x + 2y = -6$
$3x + y = 3$
one

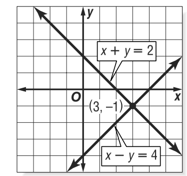
7-1 Study Guide and Intervention *(continued)*

Graphing Systems of Equations

Solve by Graphing One method of solving a system of equations is to graph the equations on the same coordinate plane.

Example **Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

a. $\boldsymbol{x + y = 2}$
$\boldsymbol{x - y = 4}$

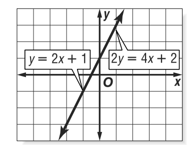


The graphs intersect. Therefore, there is one solution. The point $(3, -1)$ seems to lie on both lines. Check this estimate by replacing x with 3 and y with -1 in each equation.

$x + y = 2$
$3 + (-1) = 2$ ✓
$x - y = 4$
$3 - (-1) = 3 + 1$ or 4 ✓

The solution is $(3, -1)$.

b. $\boldsymbol{y = 2x + 1}$
$\boldsymbol{2y = 4x + 2}$

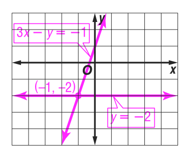


The graphs coincide. Therefore there are infinitely many solutions.

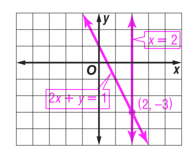
Exercises

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

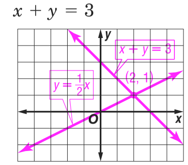
1. $y = -2$ one; $(-1, -2)$
$3x - y = -1$



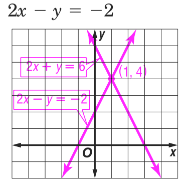
2. $x = 2$ one; $(2, -3)$
$2x + y = 1$



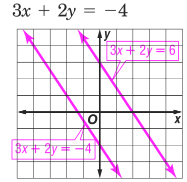
3. $y = \frac{1}{2}x$ one; $(2, 1)$
$x + y = 3$



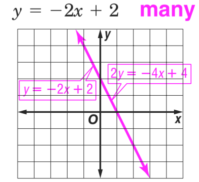
4. $2x + y = 6$ one; $(1, 4)$
$2x - y = -2$



5. $3x + 2y = 6$ no solution
$3x + 2y = -4$



6. $2y = -4x + 4$ infinitely many
$y = -2x + 2$

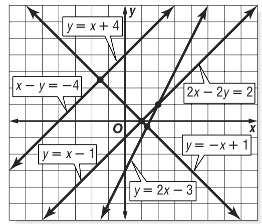


NAME ______________________ DATE __________ PERIOD _____

7-1 Skills Practice

Graphing Systems of Equations

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



1. $y = x - 1$
$y = -x + 1$ **one**

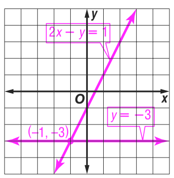
2. $x - y = -4$
$y = x + 4$ **infinitely many**

3. $y = x + 4$
$2x - 2y = 2$
no solution

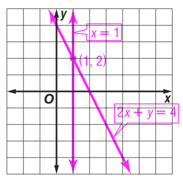
4. $y = 2x - 3$
$2x - 2y = 2$
one

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

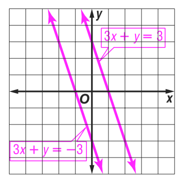
5. $2x - y = 1$
$y = -3$ **one; (−1, −3)**



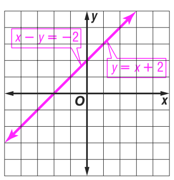
6. $x = 1$
$2x + y = 4$ **one; (1, 2)**



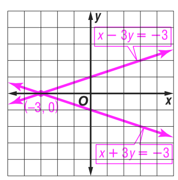
7. $3x + y = -3$
$3x + y = 3$ **no solution**



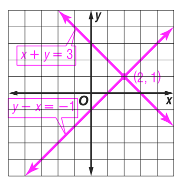
8. $y = x + 2$
$x - y = -2$ **infinitely many**



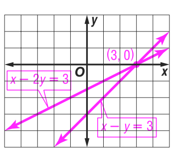
9. $x + 3y = -3$
$x - 3y = -3$ **one; (−3, 0)**



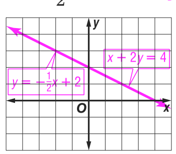
10. $y - x = -1$
$x + y = 3$ **one; (2, 1)**



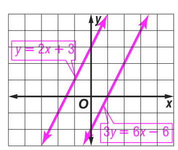
11. $x - y = 3$
$x - 2y = 3$ **one; (3, 0)**



12. $x + 2y = 4$
$y = -\frac{1}{2}x + 2$ **infinitely many**



13. $y = 2x + 3$
$3y = 6x - 6$ **no solution**



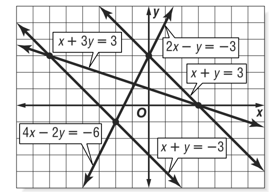
Lesson 7-1

NAME ______________________ DATE __________ PERIOD _____

7-1 Practice (Average)

Graphing Systems of Equations

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



1. $x + y = 3$
$x + y = -3$
no solution

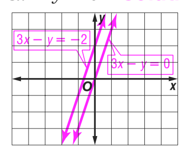
2. $2x - y = -3$
$4x - 2y = -6$
infinitely many

3. $x + 3y = 3$
$x + y = -3$ **one**

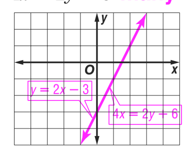
4. $x + 3y = 3$
$2x - y = -3$ **one**

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

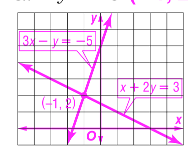
5. $3x - y = -2$
$3x - y = 0$ **no solution**



6. $y = 2x - 3$
$4x = 2y + 6$ **infinitely many**



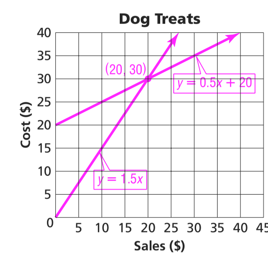
7. $x + 2y = 3$
$3x - y = -5$ **one; (−1, 2)**



BUSINESS For Exercises 8 and 9, use the following information.

Nick plans to start a home-based business producing and selling gourmet dog treats. He figures it will cost $20 in operating costs per week plus $0.50 to produce each treat. He plans to sell each treat for $1.50.

8. Graph the system of equations $y = 0.5x + 20$ and $y = 1.5x$ to represent the situation.



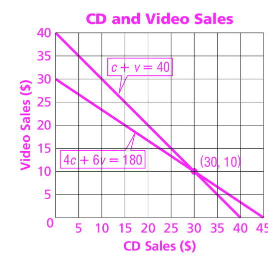
9. How many treats does Nick need to sell per week to break even? **20**

SALES For Exercises 10–12, use the following information.

A used book store also started selling used CDs and videos. In the first week, the store sold 40 used CDs and videos, at $4.00 per CD and $6.00 per video. The sales for both CDs and videos totaled $180.00

10. Write a system of equations to represent the situation. **$c + v = 40$ $4c + 6v = 180$**

11. Graph the system of equations.



12. How many CDs and videos did the store sell in the first week? **30 CDs and 10 videos**

Answers

NAME ______________________ DATE __________ PERIOD _____

7-1 Reading to Learn Mathematics

Graphing Systems of Equations

Pre-Activity **How can you use graphs to compare the sales of two products?**

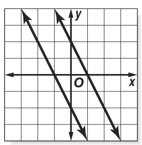
Read the introduction to Lesson 7-1 at the top of page 369 in your textbook.

- What is meant by the term linear function? **Sample answer: a function whose graph is a line**
- What does it mean to say that two lines intersect? **Sample answer: The lines cross.**

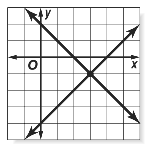
Reading the Lesson

1. Each figure shows the graph of a system of two equations. Write the letter of the figures that illustrate each statement.

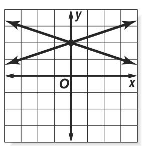
A.



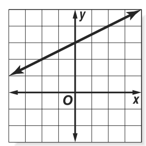
B.



C.



D.



a. A system of two linear equations can have an infinite number of solutions. **D**

b. A system of equations is consistent if there is at least one ordered pair that satisfies both equations. **B, C, D**

c. If two graphs are parallel, there are no ordered pairs that satisfy both equations. **A**

d. If a system of equations has exactly one solution, it is independent. **B, C**

e. If a system of equations has an infinite number of solutions, it is dependent. **D**

Helping You Remember

2. Describe how you can solve a system of equations by graphing. **Sample answer: Graph the equations on the same coordinate plane. Locate any points of intersection.**

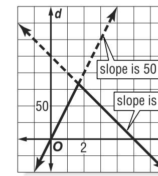
Lesson 7-1

NAME ______________________ DATE __________ PERIOD _____

7-1 Enrichment

Graphing a Trip

The distance formula, $d = rt$, is used to solve many types of problems. If you graph an equation such as $d = 50t$, the graph is a model for a car going at 50 mi/h. The time the car travels is t; the distance in miles the car covers is d. The slope of the line is the speed.



Suppose you drive to a nearby town and return. You average 50 mi/h on the trip out but only 25 mi/h on the trip home. The round trip takes 5 hours. How far away is the town?

The graph at the right represents your trip. Notice that the return trip is shown with a negative slope because you are driving in the opposite direction.

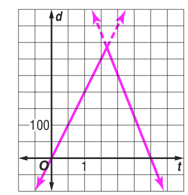
Solve each problem.

1. Estimate the answer to the problem in the above example. About how far away is the town?

about 80 miles

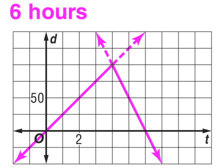
2. Graph this trip and solve the problem. An airplane has enough fuel for 3 hours of safe flying. On the trip out the pilot averages 200 mi/h flying against a headwind. On the trip back, the pilot averages 250 mi/h. How long a trip out can the pilot make?

about $1\frac{2}{3}$ hours and 330 miles



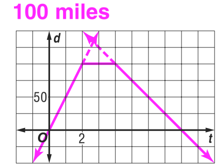
3. Graph this trip and solve the problem. You drive to a town 100 miles away. On the trip out you average 25 mi/h. On the trip back you average 50 mi/h. How many hours do you spend driving?

6 hours



4. Graph this trip and solve the problem. You drive at an average speed of 50 mi/h to a discount shopping plaza, spend 2 hours shopping, and then return at an average speed of 25 mi/h. The entire trip takes 8 hours. How far away is the shopping plaza?

100 miles



NAME ______________ DATE ________ PERIOD _____

7-2 Study Guide and Intervention
Substitution

Substitution One method of solving systems of equations is **substitution**.

Example 1 **Use substitution to solve the system of equations.**
$y = 2x$
$4x - y = -4$

Substitute $2x$ for y in the second equation.

$4x - y = -4$ Second equation
$4x - 2x = -4$ $y = 2x$
$2x = -4$ Combine like terms.
$\frac{2x}{2} = \frac{-4}{2}$ Divide each side by 2.
$x = -2$ Simplify.

Use $y = 2x$ to find the value of y.

$y = 2x$ First equation
$y = 2(-2)$ $x = -2$
$y = -4$ Simplify.

The solution is $(-2, -4)$.

Example 2 **Solve for one variable, then substitute.**
$x + 3y = 7$
$2x - 4y = -6$

Solve the first equation for x since the coefficient of x is 1.

$x + 3y = 7$ First equation
$x + 3y - 3y = 7 - 3y$ Subtract $3y$ from each side.
$x = 7 - 3y$ Simplify.

Find the value of y by substituting $7 - 3y$ for x in the second equation.

$2x - 4y = -6$ Second equation
$2(7 - 3y) - 4y = -6$ $x = 7 - 3y$
$14 - 6y - 4y = -6$ Distributive Property
$14 - 10y = -6$ Combine like terms.
$14 - 10y - 14 = -6 - 14$ Subtract 14 from each side.
$-10y = -20$ Simplify.
$\frac{-10y}{-10} = \frac{-20}{-10}$ Divide each side by -10.
$y = 2$ Simplify.

Use $y = 2$ to find the value of x.

$x = 7 - 3y$
$x = 7 - 3(2)$
$x = 1$

The solution is $(1, 2)$.

Exercises

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

1. $y = 4x$; $3x - y = 1$ **(−1, −4)**

2. $x = 2y$; $y = x - 2$ **(4, 2)**

3. $x = 2y - 3$; $x = 2y + 4$ **no solution**

4. $x - 2y = -1$; $3y = x + 4$ **(5, 3)**

5. $c - 4d = 1$; $2c - 8d = 2$ **infinitely many**

6. $x + 2y = 0$; $3x + 4y = 4$ **(4, −2)**

7. $2b = 6a - 14$; $3a - b = 7$ **infinitely many**

8. $x + y = 16$; $2y = -2x + 2$ **no solution**

9. $y = -x + 3$; $2y + 2x = 4$ **no solution**

10. $x = 2y$; $0.25x + 0.5y = 10$ **(20, 10)**

11. $x - 2y = -5$; $x + 2y = -1$ **(−3, 1)**

12. $-0.2x + y = 0.5$; $0.4x + y = 1.1$ **(1, 0.7)**

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7-2 Study Guide and Intervention *(continued)*
Substitution

Real-World Problems Substitution can also be used to solve real-world problems involving systems of equations. It may be helpful to use tables, charts, diagrams, or graphs to help you organize data.

Example **CHEMISTRY How much of a 10% saline solution should be mixed with a 20% saline solution to obtain 1000 milliliters of a 12% saline solution?**

Let s = the number of milliliters of 10% saline solution.
Let t = the number of milliliters of 20% saline solution.
Use a table to organize the information.

	10% saline	20% saline	12% saline
Total milliliters	s	t	1000
Milliliters of saline	$0.10s$	$0.20t$	0.12(1000)

Write a system of equations.
$s + t = 1000$
$0.10s + 0.20t = 0.12(1000)$
Use substitution to solve this system.

$s + t = 1000$ First equation
$s = 1000 - t$ Solve for s.
$0.10s + 0.20t = 0.12(1000)$ Second equation
$0.10(1000 - t) + 0.20t = 0.12(1000)$ $s = 1000 - t$
$100 - 0.10t + 0.20t = 0.12(1000)$ Distributive Property
$100 + 0.10t = 0.12(1000)$ Combine like terms.
$0.10t = 20$ Simplify.
$\frac{0.10t}{0.10} = \frac{20}{0.10}$ Divide each side by 0.10.
$t = 200$ Simplify.
$s + t = 1000$ First equation
$s + 200 = 1000$ $t = 200$
$s = 800$ Solve for s.

800 milliliters of 10% solution and 200 milliliters of 20% solution should be used.

Exercises

1. SPORTS At the end of the 2000-2001 football season, 31 Super Bowl games had been played with the current two football leagues, the American Football Conference (AFC) and the National Football Conference (NFC). The NFC won five more games than the AFC. How many games did each conference win? **Source:** *New York Times Almanac* **AFC 13; NFC 18**

2. CHEMISTRY A lab needs to make 100 gallons of an 18% acid solution by mixing a 12% acid solution with a 20% solution. How many gallons of each solution are needed? **25 gal of 12% solution and 75 gal of 20% solution**

3. GEOMETRY The perimeter of a triangle is 24 inches. The longest side is 4 inches longer than the shortest side, and the shortest side is three-fourths the length of the middle side. Find the length of each side of the triangle. **6 in., 8 in., 10 in.**

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7-2 Skills Practice

Substitution

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

1. $y = 4x$
   $x + y = 5$ **(1, 4)**
2. $y = 2x$
   $x + 3y = -14$ **(−2, −4)**
3. $y = 3x$
   $2x + y = 15$ **(3, 9)**
4. $x = -4y$
   $3x + 2y = 20$ **(8, −2)**
5. $y = x - 1$
   $x + y = 3$ **(2, 1)**
6. $x = y - 7$
   $x + 8y = 2$ **(−6, 1)**
7. $y = 4x - 1$
   $y = 2x - 5$ **(−2, −9)**
8. $y = 3x + 8$
   $5x + 2y = 5$ **(−1, 5)**
9. $2x - 3y = 21$
   $y = 3 - x$ **(6, −3)**
10. $y = 5x - 8$
    $4x + 3y = 33$ **(3, 7)**
11. $x + 2y = 13$
    $3x - 5y = 6$ **(7, 3)**
12. $x + 5y = 4$
    $3x + 15y = -1$ **no solution**
13. $3x - y = 4$
    $2x - 3y = -9$ **(3, 5)**
14. $x + 4y = 8$
    $2x - 5y = 29$ **(12, −1)**
15. $x - 5y = 10$
    $2x - 10y = 20$ **infinitely many**
16. $5x - 2y = 14$
    $2x - y = 5$ **(4, 3)**
17. $2x + 5y = 38$
    $x - 3y = -3$ **(9, 4)**
18. $x - 4y = 27$
    $3x + y = -23$ **(−5, −8)**
19. $2x + 2y = 7$
    $x - 2y = -1$ $\left(2, \frac{3}{2}\right)$
20. $2.5x + y = -2$
    $3x + 2y = 0$ **(−2, 3)**

Lesson 7-2

NAME ________________ DATE ________ PERIOD ____

7-2 Practice (Average)

Substitution

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

1. $y = 6x$
   $2x + 3y = -20$ **(−1, −6)**
2. $x = 3y$
   $3x - 5y = 12$ **(9, 3)**
3. $x = 2y + 7$
   $x = y + 4$ **(1, −3)**
4. $y = 2x - 2$
   $y = x + 2$ **(4, 6)**
5. $y = 2x + 6$
   $2x - y = 2$ **no solution**
6. $3x + y = 12$
   $y = -x - 2$ **(7, −9)**
7. $x + 2y = 13$
   $-2x - 3y = -18$ **(−3, 8)**
8. $x - 2y = 3$
   $4x - 8y = 12$ **infinitely many**
9. $x - 5y = 36$
   $2x + y = -16$ **(−4, −8)**
10. $2x - 3y = -24$
    $x + 6y = 18$ **(−6, 4)**
11. $x + 14y = 84$
    $2x - 7y = -7$ **(14, 5)**
12. $0.3x - 0.2y = 0.5$
    $x - 2y = -5$ **(5, 5)**
13. $0.5x + 4y = -1$
    $x + 2.5y = 3.5$ **(6, −1)**
14. $3x - 2y = 11$
    $x - \frac{1}{2}y = 4$ **(5, 2)**
15. $\frac{1}{2}x + 2y = 12$
    $x - 2y = 6$ **(12, 3)**
16. $\frac{1}{3}x - y = 3$
    $2x + y = 25$
    (12, 1)
17. $4x - 5y = -7$
    $y = 5x$
    $\left(\frac{1}{3}, 1\frac{2}{3}\right)$
18. $x - 3y = -4$
    $2x + 6y = 5$
    $\left(-\frac{3}{4}, 1\frac{1}{12}\right)$

EMPLOYMENT For Exercises 19–21, use the following information.

Kenisha sells athletic shoes part-time at a department store. She can earn either \$500 per month plus a 4% commission on her total sales, or \$400 per month plus a 5% commission on total sales.

19. Write a system of equations to represent the situation.
    $y = 0.04x + 500$ **and** $y = 0.05x + 400$
20. What is the total price of the athletic shoes Kenisha needs to sell to earn the same income from each pay scale? **\$10,000**
21. Which is the better offer? **the first offer if she expects to sell less than \$10,000 in shoes, and the second offer if she expects to sell more than \$10,000 in shoes**

MOVIE TICKETS For Exercises 22 and 23, use the following information.

Tickets to a movie cost \$7.25 for adults and \$5.50 for students. A group of friends purchased 8 tickets for \$52.75.

22. Write a system of equations to represent the situation.
    $x + y = 8$ **and** $7.25x + 5.5y = 52.75$
23. How many adult tickets and student tickets were purchased? **5 adult and 3 student**

NAME ______________________ DATE __________ PERIOD _____

7-2 Reading to Learn Mathematics

Substitution

Pre-Activity **How can a system of equations be used to predict media use?**

Read the introduction to Lesson 7-2 at the top of page 376 in your textbook.

- What is the system of equations? **$y = -2.8x + 170$, $y = 14.4x + 2$**
- Based on the graph, are there 0, 1, or infinitely many solutions of the system? **1 solution**

Reading the Lesson

1. Describe how you would use substitution to solve each system of equations.

a. $y = -2x$
$x + 3y = 15$
Substitute $-2x$ for y in the second equation. Simplify and solve for x. Then use the value of x to find the value of y.

b. $3x - 2y = 12$
$x = 2y$
Substitute $2y$ for x in the first equation. Simplify and solve for y. Then use the value of y to find the value of x.

c. $x + 2y = 7$
$2x - 8y = 8$
Solve the first equation for x. Substitute the expression you find for x in the second equation. Simplify and solve for y. Then use the value of y to find the value of x.

d. $-3x + 5y = 81$
$2x + y = 24$
Solve the second equation for y. Substitute the expression you find for y in the first equation. Simplify and solve for x. Then use the value of x to find the value of y.

2. Jess solved a system of equations and her result was $-8 = -8$. All of her work was correct. Describe the graph of the system. Explain. **It is a line. There are infinitely many solutions, since $-8 = -8$ is always true.**

3. Miguel solved a system of equations and his result was $5 = -2$. All of his work was correct. Describe the graph of the system. Explain. **The graph has two parallel lines, since $5 = -2$ is always false.**

Helping You Remember

4. What is usually the first step in solving a system of equations by substitution? **Sample answer: Solve one of the equations for one variable in terms of the other.**

Lesson 7-2

NAME ______________________ DATE __________ PERIOD _____

7-2 Enrichment

Equations of Lines and Planes in Intercept Form

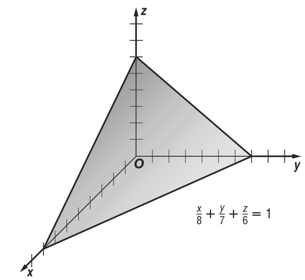
One form that a linear equation may take is **intercept form**. The constants a and b are the x- and y-intercepts of the graph.

$$\frac{x}{a} + \frac{y}{b} = 1$$

In three-dimensional space, the equation of a plane takes a similar form.

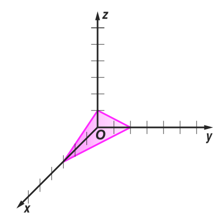
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here, the constants a, b, and c are the points where the plane meets the x, y, and z-axes.



Solve each problem.

1. Graph the equation $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 1$.

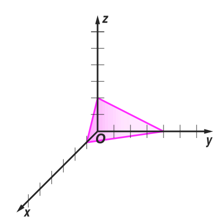


2. For the plane in Exercise 1, write an equation for the line where the plane intersects the xy-plane. Use intercept forms.
$\underline{x} + \underline{y} = 1$

3. Write an equation for the line where the plane intersects the xz-plane.
$\underline{x} + \underline{z} = 1$

4. Write an equation for the line where the plane intersects the yz-plane.
$\underline{y} + \underline{z} = 1$

5. Graph the equation $\frac{x}{1} + \frac{y}{4} + \frac{z}{2} = 1$.



6. Write an equation for the xy-plane.
$z = 0$

7. Write an equation for the yz-plane.
$x = 0$

8. Write an equation for a plane parallel to the xy-plane with a z-intercept of 2.
$z = 2$

9. Write an equation for a plane parallel to the yz-plane with an x-intercept of 23.
$x = -3$

Answers

NAME ______________________ DATE ____________ PERIOD _____

7-3 Study Guide and Intervention

Elimination Using Addition and Subtraction

Elimination Using Addition In systems of equations in which the coefficients of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called **elimination**.

Example 1 **Use addition to solve the system of equations.**
$\boldsymbol{x - 3y = 7}$
$\boldsymbol{3x + 3y = 9}$

Write the equations in column form and add to eliminate y.

$$\begin{array}{r} x - 3y = 7 \\ (+)\ 3x + 3y = 9 \\ \hline 4x \quad\quad = 16 \end{array}$$

Solve for x.

$$\frac{4x}{4} = \frac{16}{4}$$
$$x = 4$$

Substitute 4 for x in either equation and solve for y.

$$4 - 3y = 7$$
$$4 - 3y - 4 = 7 - 4$$
$$-3y = 3$$
$$\frac{-3y}{-3} = \frac{3}{-3}$$
$$y = -1$$

The solution is $(4, -1)$.

Example 2 **The sum of two numbers is 70 and their difference is 24. Find the numbers.**

Let x represent one number and y represent the other number.

$$\begin{array}{r} x + y = 70 \\ (+)\ x - y = 24 \\ \hline 2x \quad\quad = 94 \end{array}$$
$$\frac{2x}{2} = \frac{94}{2}$$
$$x = 47$$

Substitute 47 for x in either equation.

$$47 + y = 70$$
$$47 + y - 47 = 70 - 47$$
$$y = 23$$

The numbers are 47 and 23.

Exercises

Use elimination to solve each system of equations.

1. $x + y = -4$; $x - y = 2$ **(−1, −3)**
2. $2m - 3n = 14$; $m + 3n = -11$ **(1, −4)**
3. $3a - b = -9$; $-3a - 2b = 0$ **(−2, 3)**
4. $-3x - 4y = -1$; $3x - y = -4$ **(−1, 1)**
5. $3c + d = 4$; $2c - d = 6$ **(2, −2)**
6. $-2x + 2y = 9$; $2x - y = -6$ $\left(-\frac{3}{2}, 3\right)$
7. $2x + 2y = -2$; $3x - 2y = 12$ **(2, −3)**
8. $4x - 2y = -1$; $-4x + 4y = -2$ $\left(-1, -\frac{3}{2}\right)$
9. $x - y = 2$; $x + y = -3$ $\left(-\frac{1}{2}, -\frac{5}{2}\right)$
10. $2x - 3y = 12$; $4x + 3y = 24$ **(6, 0)**
11. $-0.2x + y = 0.5$; $0.2x + 2y = 1.6$ **(1, 0.7)**
12. $0.1x + 0.3y = 0.9$; $0.1x - 0.3y = 0.2$ $\left(\frac{11}{2}, \frac{7}{6}\right)$

13. Rema is older than Ken. The difference of their ages is 12 and the sum of their ages is 50. Find the age of each. **Rema is 31 and Ken is 19.**

14. The sum of the digits of a two-digit number is 12. The difference of the digits is 2. Find the number if the units digit is larger than the tens digit. **57**

Lesson 7-3

NAME ______________________ DATE ____________ PERIOD _____

7-3 Study Guide and Intervention *(continued)*

Elimination Using Addition and Subtraction

Elimination Using Subtraction In systems of equations where the coefficients of the x or y terms are the same, solve the system by subtracting the equations.

Example **Use subtraction to solve the system of equations.**
$\boldsymbol{2x - 3y = 11}$
$\boldsymbol{5x - 3y = 14}$

$2x - 3y = 11$ $(-)\ 5x - 3y = 14$	Write the equations in column form and subtract.
$-3x = -3$	Subtract the two equations. y is eliminated.
$\frac{-3x}{-3} = \frac{-3}{-3}$	Divide each side by −3.
$x = 1$	Simplify.
$2(1) - 3y = 11$	Substitute 1 for x in either equation.
$2 - 3y = 11$	Simplify.
$2 - 3y - 2 = 11 - 2$	Subtract 2 from each side.
$-3y = 9$	Simplify.
$\frac{-3y}{-3} = \frac{9}{-3}$	Divide each side by −3.
$y = -3$	Simplify.

The solution is $(1, -3)$.

Exercises

Use elimination to solve each system of equations.

1. $6x + 5y = 4$; $6x - 7y = -20$ **(−1, 2)**
2. $3m - 4n = -14$; $3m + 2n = -2$ **(−2, 2)**
3. $3a + b = 1$; $a + b = 3$ **(−1, 4)**
4. $-3x - 4y = -23$; $-3x + y = 2$ **(1, 5)**
5. $c - 3d = 11$; $2c - 3d = 16$ **(5, −2)**
6. $x - 2y = 6$; $x + y = 3$ **(4, −1)**
7. $2a - 3b = -13$; $2a + 2b = 7$ $\left(-\frac{1}{2}, 4\right)$
8. $4x + 2y = 6$; $4x + 4y = 10$ $\left(\frac{1}{2}, 2\right)$
9. $5s - t = 6$; $5s + 2t = 3$ **(1, −1)**
10. $6x - 3y = 12$; $4x - 3y = 24$ **(−6, −16)**
11. $x + 2y = 3.5$; $x - 3y = -9$ **(−1.5, 2.5)**
12. $0.2x + y = 0.7$; $0.2x + 2y = 1.2$ **(1, 0.5)**

13. The sum of two numbers is 70. One number is ten more than twice the other number. Find the numbers. **50; 20**

14. **GEOMETRY** Two angles are supplementary. The measure of one angle is 10° more than three times the other. Find the measure of each angle. **42.5°; 137.5°**

NAME ________________ DATE ________ PERIOD ____

7-3 Skills Practice

Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

1. $x - y = 1$
$x + y = 3$ **(2, 1)**

2. $-x + y = 1$
$x + y = 11$ **(5, 6)**

3. $x + 4y = 11$
$x - 6y = 11$ **(11, 0)**

4. $-x + 3y = 6$
$x + 3y = 18$ **(6, 4)**

5. $3x + 4y = 19$
$3x + 6y = 33$ **(−3, 7)**

6. $x + 4y = -8$
$x - 4y = -8$ **(−8, 0)**

7. $3a + 4b = 2$
$4a - 4b = 12$ **(2, −1)**

8. $3c - d = -1$
$-3c - d = 5$ **(−1, −2)**

9. $2x - 3y = 9$
$-5x - 3y = 30$ **(−3, −5)**

10. $x - y = 4$
$2x + y = -4$ **(0, −4)**

11. $3m - n = 26$
$-2m - n = -24$ **(10, 4)**

12. $5x - y = -6$
$-x + y = 2$ **(−1, 1)**

13. $6x - 2y = 32$
$4x - 2y = 18$ **(7, 5)**

14. $3x + 2y = -19$
$-3x - 5y = 25$ **(−5, −2)**

15. $7m + 4n = 2$
$7m + 2n = 8$ **(2, −3)**

16. $2x - 5y = -28$
$4x + 5y = 4$ **(−4, 4)**

17. The sum of two numbers is 28 and their difference is 4. What are the numbers? **12, 16**

18. Find the two numbers whose sum is 29 and whose difference is 15. **7, 22**

19. The sum of two numbers is 24 and their difference is 2. What are the numbers? **13, 11**

20. Find the two numbers whose sum is 54 and whose difference is 4. **25, 29**

21. Two times a number added to another number is 25. Three times the first number minus the other number is 20. Find the numbers. **9, 7**

Lesson 7-3

NAME ________________ DATE ________ PERIOD ____

7-3 Practice (Average)

Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

1. $x - y = 1$
$x + y = -9$
(−4, −5)

2. $p + q = -2$
$p - q = 8$
(3, −5)

3. $4x + y = 23$
$3x - y = 12$
(5, 3)

4. $2x + 5y = -3$
$2x + 2y = 6$
(6, −3)

5. $3x + 2y = -1$
$4x + 2y = -6$
(−5, 7)

6. $5x + 3y = 22$
$5x - 2y = 2$
(2, 4)

7. $5x + 2y = 7$
$-2x + 2y = -14$
(3, −4)

8. $3x - 9y = -12$
$3x - 15y = -6$
(−7, −1)

9. $-4c - 2d = -2$
$2c - 2d = -14$
(−2, 5)

10. $2x - 6y = 6$
$2x + 3y = 24$
(9, 2)

11. $7x + 2y = 2$
$7x - 2y = -30$
(−2, 8)

12. $4.25x - 1.28y = -9.2$
$x + 1.28y = 17.6$
(1.6, 12.5)

13. $2x + 4y = 10$
$x - 4y = -2.5$
(2.5, 1.25)

14. $2.5x + y = 10.7$
$2.5x + 2y = 12.9$
(3.4, 2.2)

15. $6m - 8n = 3$
$2m - 8n = -3$
$\left(1\frac{1}{2}, \frac{3}{4}\right)$

16. $4a + b = 2$
$4a + 3b = 10$
$\left(-\frac{1}{2}, 4\right)$

17. $-\frac{1}{3}x - \frac{4}{3}y = -2$
$\frac{1}{3}x - \frac{2}{3}y = 4$
(10, −1)

18. $\frac{3}{4}x - \frac{1}{2}y = 8$
$\frac{3}{2}x + \frac{1}{2}y = 19$
(12, 2)

19. The sum of two numbers is 41 and their difference is 5. What are the numbers? **18, 23**

20. Four times one number added to another number is 36. Three times the first number minus the other number is 20. Find the numbers. **8, 4**

21. One number added to three times another number is 24. Five times the first number added to three times the other number is 36. Find the numbers. **3, 7**

22. LANGUAGES English is spoken as the first or primary language in 78 more countries than Farsi is spoken as the first language. Together, English and Farsi are spoken as a first language in 130 countries. In how many countries is English spoken as the first language? In how many countries is Farsi spoken as the first language?
English: 104 countries, Farsi: 26 countries

23. DISCOUNTS At a sale on winter clothing, Cody bought two pairs of gloves and four hats for \$43.00. Tori bought two pairs of gloves and two hats for \$30.00. What were the prices for the gloves and hats? **gloves: \$8.50, hats: \$6.50.**

Answers

NAME ______________________ DATE __________ PERIOD _____

7-3 Reading to Learn Mathematics

Elimination Using Addition and Subtraction

Pre-Activity **How can you use a system of equations to solve problems about weather?**

Read the introduction to Lesson 7-3 at the top of page 382 in your textbook.

What fact explains why the variable d gets eliminated from the system of equations? **$d - d = 0$**

Reading the Lesson

1. Write *addition* or *subtraction* to tell which operation it would be easiest to use to eliminate a variable of the system. Explain your choice.

	System of Equations	Operation	Explanation
a.	$3x + 5y = 12$ $-3x + 2y = 6$	addition	The coefficients of the x terms are additive inverses.
b.	$3x + 5y = 7$ $3x - 2y = 8$	subtraction	The coefficients of the x terms are the same.
c.	$-x - 4y = 9$ $4x - 4y = 6$	subtraction	The coefficients of the y terms are the same.
d.	$5x - 7y = 17$ $8x + 7y = 9$	addition	The coefficients of the y terms are additive inverses.

Helping You Remember

2. Tell how you can decide whether to use addition or subtraction to eliminate a variable in a system of equations. **Sample answer: Look at the coefficients of each variable. If the coefficients of one of the variables are additive inverses, you can use addition to eliminate the variable. If the coefficients of one variable are the same, you can use subtraction to eliminate the variable.**

Lesson 7-3

NAME ______________________ DATE __________ PERIOD _____

7-3 Enrichment

Rózsa Péter

Rózsa Péter (1905–1977) was a Hungarian mathematician dedicated to teaching others about mathematics. As professor of mathematics at a teachers' college in Budapest, she wrote several mathematics textbooks and championed reforms in the teaching of mathematics. In 1945 she wrote *Playing with Infinity: Mathematical Explorations and Excursions*, a popular work in which she attempted to convey the spirit of mathematics to the general public.

By far Péter's greatest contribution to mathematics was her pioneering research in the field of recursive function theory. When you evaluate a function *recursively*, you begin with one initial value of x. Working from this single number, you can use the function to generate an entire sequence of numbers. For instance, here is how you use an initial value of $x = 1$ to evaluate the function $f(x) = 3x$ recursively.

$f(1) = 3(1) = 3$
$\downarrow$
$f(3) = 3(3) = 9$
$\downarrow$
$f(9) = 3(9) = 27$
$\downarrow$
$f(27) = 3(27) = 81$
$\downarrow$
$f(81) = 3(81) = 243$
$\downarrow$

The first five numbers of the sequence generated by this function are 3, 9, 27, 81, and 243.

Write the first five numbers of the sequence generated by each function, using the given number as the initial value of x.

1. $f(x) = 3x; x = 2$
6, 18, 54, 162, 486

2. $g(x) = x - 5; x = 1$
−4, −9, −14, −19, −24

3. $f(x) = 2x + 1; x = -3$
−5, −9, −17, −33, −65

4. $f(x) = x^2; x = 2$
4; 16; 256; 65,536; 4,294,967,296

5. $h(x) = -x; x = 3$
−3, 3, −3, 3, −3

6. $k(x) = \frac{1}{x}; x = 10$
$\frac{1}{10}$, 10, $\frac{1}{10}$, 10, $\frac{1}{10}$

NAME ______________ DATE ________ PERIOD _____

7-4 Study Guide and Intervention

Elimination Using Multiplication

Elimination Using Multiplication Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

Example 1 **Use elimination to solve the system of equations.**

$\boldsymbol{x + 10y = 3}$
$\boldsymbol{4x + 5y = 5}$

If you multiply the second equation by -2, you can eliminate the y terms.

$$\begin{aligned} x + 10y &= 3 \\ (+)\ -8x - 10y &= -10 \\ \hline -7x \quad &= -7 \\ \frac{-7x}{-7} &= \frac{-7}{-7} \\ x &= 1 \end{aligned}$$

Substitute 1 for x in either equation.

$$\begin{aligned} 1 + 10y &= 3 \\ 1 + 10y - 1 &= 3 - 1 \\ 10y &= 2 \\ \frac{10y}{10} &= \frac{2}{10} \\ y &= \frac{1}{5} \end{aligned}$$

The solution is $\left(1, \frac{1}{5}\right)$.

Example 2 **Use elimination to solve the system of equations.**

$\boldsymbol{3x - 2y = -7}$
$\boldsymbol{2x - 5y = 10}$

If you multiply the first equation by 2 and the second equation by -3, you can eliminate the x terms.

$$\begin{aligned} 6x - 4y &= -14 \\ (+)\ -6x + 15y &= -30 \\ \hline 11y &= -44 \\ \frac{11y}{11} &= \frac{-44}{11} \\ y &= -4 \end{aligned}$$

Substitute -4 for y in either equation.

$$\begin{aligned} 3x - 2(-4) &= -7 \\ 3x + 8 &= -7 \\ 3x + 8 - 8 &= -7 - 8 \\ 3x &= -15 \\ \frac{3x}{3} &= \frac{-15}{3} \\ x &= -5 \end{aligned}$$

The solution is $(-5, -4)$.

Exercises

Use elimination to solve each system of equations.

1. $2x + 3y = 6$
$x + 2y = 5$ **$(-3, 4)$**

2. $2m + 3n = 4$
$-m + 2n = 5$ **$(-1, 2)$**

3. $3a - b = 2$
$a + 2b = 3$ **$(1, 1)$**

4. $4x + 5y = 6$
$6x - 7y = -20$ **$(-1, 2)$**

5. $4c - 3d = 22$
$2c - d = 10$ **$(4, -2)$**

6. $3x - 4y = -4$
$x + 3y = -10$ **$(-4, -2)$**

7. $4s - t = 9$
$5s + 2t = 8$ **$(2, -1)$**

8. $4a - 3b = -8$
$2a + 2b = 3$ **$\left(-\frac{1}{2}, 2\right)$**

9. $2x + 2y = 5$
$4x - 4y = 10$ **$\left(\frac{5}{2}, 0\right)$**

10. $6x - 4y = -8$
$4x + 2y = -3$ **$\left(-1, \frac{1}{2}\right)$**

11. $4x + 2y = -5$
$-2x - 4y = 1$ **$\left(-\frac{3}{2}, \frac{1}{2}\right)$**

12. $2x + y = 3.5$
$-x + 2y = 2.5$ **$(0.9, 1.7)$**

13. GARDENING The length of Sally's garden is 4 meters greater than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally's garden? **28 m by 8 m**

14. Anita is $4\frac{1}{2}$ years older than Basilio. Three times Anita's age added to six times Basilio's age is 36. How old are Anita and Basilio? **Anita: 7 yr; Basilio: $2\frac{1}{2}$ yr**

Lesson 7-4

NAME ______________ DATE ________ PERIOD _____

7-4 Study Guide and Intervention *(continued)*

Elimination Using Multiplication

Determine the Best Method The methods to use for solving systems of linear equations are summarized in the table below.

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or −1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example **Determine the best method to solve the system of equations. Then solve the system.**

$\boldsymbol{6x + 2y = 20}$
$\boldsymbol{-2x + 4y = -16}$

Since the coefficients of x will be additive inverses of each other if you multiply the second equation by 3, use elimination.

$$\begin{aligned} 6x + 2y &= 20 \\ (+)\ -6x + 12y &= -48 && \text{Multiply the second equation by 3.} \\ \hline 14y &= -28 && \text{Add the two equations. } x \text{ is eliminated.} \\ \frac{14y}{14} &= \frac{-28}{14} && \text{Divide each side by 14.} \\ y &= -2 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} 6x + 2(-2) &= 20 && \text{Substitute } -2 \text{ for } y \text{ in either equation.} \\ 6x - 4 &= 20 && \text{Simplify.} \\ 6x - 4 + 4 &= 20 + 4 && \text{Add 4 to each side.} \\ 6x &= 24 && \text{Simplify.} \\ \frac{6x}{6} &= \frac{24}{6} && \text{Divide each side by 6.} \\ x &= 4 && \text{Simplify.} \end{aligned}$$

The solution is $(4, -2)$.

Exercises

Determine the best method to solve each system of equations. Then solve the system.

1. $x + 2y = 3$
$x + y = 1$
elimination (−); $(-1, 2)$

2. $m + 6n = -8$
$m = 2n + 8$
substitution; $(4, -2)$

3. $a - b = 6$
$a = 2b + 7$
substitution; $(5, -1)$

4. $4x + y = 15$
$-x - 3y = -12$
substitution; $(3, 3)$

5. $3c - d = 14$
$c - d = 2$
elimination (−); $(6, 4)$

6. $x + 2y = -9$
$y = 4x$
substitution; $(-1, -4)$

7. $4x = 2y - 10$
$x + 2y = 5$
substitution; $(-1, 3)$

8. $x = -2y$
$4x + 4y = -10$
substitution; $\left(-5, \frac{5}{2}\right)$

9. $2s - 3t = 42$
$3s + 2t = 24$
elimination (×); $(12, -6)$

10. $4a - 4b = -10$
$2a + 4b = -2$
elimination (+); $\left(-2, \frac{1}{2}\right)$

11. $4x + 10y = -6$
$-2x - 10y = 2$
elimination (+); $\left(-2, \frac{1}{5}\right)$

12. $2x = y - 3$
$-x + y = 0$
substitution; $(-3, -3)$

Answers

NAME ______________________ DATE ________ PERIOD _____

7-4 Skills Practice

Elimination Using Multiplication

Use elimination to solve each system of equations.

1. $x + y = -9$
$5x - 2y = 32$ **(2, −11)**

2. $3x + 2y = -9$
$x - y = -13$ **(−7, 6)**

3. $2x + 5y = 3$
$-x + 3y = -7$ **(4, −1)**

4. $2x + y = 3$
$-4x - 4y = -8$ **(1, 1)**

5. $4x - 2y = -14$
$3x - y = -8$ **(−1, 5)**

6. $2x + y = 0$
$5x + 3y = 2$ **(−2, 4)**

7. $5x + 3y = -10$
$3x + 5y = -6$ **(−2, 0)**

8. $2x + 3y = 14$
$3x - 4y = 4$ **(4, 2)**

9. $2x - 3y = 21$
$5x - 2y = 25$ **(3, −5)**

10. $3x + 2y = -26$
$4x - 5y = -4$ **(−6, −4)**

11. $3x - 6y = -3$
$2x + 4y = 30$ **(7, 4)**

12. $5x + 2y = -3$
$3x + 3y = 9$ **(−3, 6)**

13. Two times a number plus three times another number equals 13. The sum of the two numbers is 7. What are the numbers? **8, −1**

14. Four times a number minus twice another number is −16. The sum of the two numbers is −1. Find the numbers. **−3, 2**

Determine the best method to solve each system of equations. Then solve the system.

15. $2x + 3y = 10$
$5x + 2y = -8$ **elimination (×); (−4, 6)**

16. $8x - 7y = 18$
$3x + 7y = 26$ **elimination (+); (4, 2)**

17. $y = 2x$
$3x + 2y = 35$ **substitution; (5, 10)**

18. $3x + y = 6$
$3x + y = 3$ **elimination (−); no solution**

19. $3x - 4y = 17$
$4x + 5y = 2$ **elimination (×); (3, −2)**

20. $y = 3x + 1$
$3x - y = -1$ **substitution; infinitely many solutions**

Lesson 7-4

NAME ______________________ DATE ________ PERIOD _____

7-4 Practice (Average)

Elimination Using Multiplication

Use elimination to solve each system of equations.

1. $2x - y = -1$
$3x - 2y = 1$
(−3, −5)

2. $5x - 2y = -10$
$3x + 6y = 66$
(2, 10)

3. $7x + 4y = -4$
$5x + 8y = 28$
(−4, 6)

4. $2x - 4y = -22$
$3x + 3y = 30$
(3, 7)

5. $3x + 2y = -9$
$5x - 3y = 4$
(−1, −3)

6. $4x - 2y = 32$
$-3x - 5y = -11$
(7, −2)

7. $3x + 4y = 27$
$5x - 3y = 16$
(5, 3)

8. $0.5x + 0.5y = -2$
$x - 0.25y = 6$
(4, −8)

9. $2x - \frac{3}{4}y = -7$
$x + \frac{1}{2}y = 0$
(−2, 4)

10. Eight times a number plus five times another number is −13. The sum of the two numbers is 1. What are the numbers? **−6, 7**

11. Two times a number plus three times another number equals 4. Three times the first number plus four times the other number is 7. Find the numbers. **5, −2**

Determine the best method to solve each system of equations. Then solve the system.

12. $5x + 7y = 3$
$2x - 7y = -38$
elimination (+); (−5, 4)

13. $7x + 2y = 2$
$2x - 3y = -28$
elimination (×); (−2, 8)

14. $-6x - 2y = 14$
$6x + 8y = -20$
elimination (+); (−2, −1)

15. $x = 2y + 6$
$\frac{1}{2}x - y = 3$
substitution; infinitely many solutions

16. $4x + 3y = -2$
$4x + 3y = 3$
elimination (−); no solution

17. $y = \frac{1}{2}x$
$\frac{5}{2}x - 2y = 9$
substitution; (6, 3)

18. FINANCE Gunther invested \$10,000 in two mutual funds. One of the funds rose 6% in one year, and the other rose 9% in one year. If Gunther's investment rose a total of \$684 in one year, how much did he invest in each mutual fund?
\$7200 in the 6% fund and \$2800 in the 9% fund

19. CANOEING Laura and Brent paddled a canoe 6 miles upstream in four hours. The return trip took three hours. Find the rate at which Laura and Brent paddled the canoe in still water. **1.75 mi/h**

20. NUMBER THEORY The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 more than the original number. Find the number. **38**

NAME ____________ DATE ______ PERIOD ____

7-4 Reading to Learn Mathematics

Elimination Using Multiplication

Pre-Activity **How can a manager use a system of equations to plan employee time?**

Read the introduction to Lesson 7-4 at the top of page 387 in your textbook.

Can the system of equations be solved by elimination with addition or subtraction? Explain. **No; neither variable has coefficients that are additive inverses or equal.**

Reading the Lesson

1. Could elimination by multiplication be used to solve the system shown below? Explain.
$3x - 5y = 15$
$-6x + 7y = 11$
Yes; you can multiply the first equation by 2 to make the coefficients of the *x* terms additive inverses.

2. Tell whether it would be easiest to use substitution, elimination by addition, elimination by subtraction, or elimination by multiplication to solve the system. Explain your choice.

	System of Equations	Solution Method	Explanation
a.	$-3x + 4y = 2$ $3x + 2y = 10$	**elimination by addition**	**The coefficients of the *x* terms are additive inverses.**
b.	$x - 2y = 0$ $5x - 4y = 8$	**substitution**	**It is easy to solve the first equation for *x*.**
c.	$6x - 5y = -18$ $2x + 10y = 27$	**elimination by multiplication**	**Sample answer: You can multiply the first equation by 2 to eliminate *y* by addition.**
d.	$-2x + 3y = 9$ $3x + 3y = 12$	**elimination by subtraction**	**The coefficients of the *y* terms are the same.**

Helping You Remember

3. If you are going to solve a system by elimination, how do you decide whether you will need to multiply one or both equations by a number? **Sample answer: If both adding the equations and subtracting the equations result in equations that still have two variables, you will need to use multiplication.**

Lesson 7-4

NAME ____________ DATE ______ PERIOD ____

7-4 Enrichment

George Washington Carver and Percy Julian

In 1990, George Washington Carver and Percy Julian became the first African Americans elected to the National Inventors Hall of Fame. Carver (1864–1943) was an agricultural scientist known worldwide for developing hundreds of uses for the peanut and the sweet potato. His work revitalized the economy of the southern United States because it was no longer dependent solely upon cotton. Julian (1898–1975) was a research chemist who became famous for inventing a method of making a synthetic cortisone from soybeans. His discovery has had many medical applications, particularly in the treatment of arthritis.

There are dozens of other African American inventors whose accomplishments are not as well known. Their inventions range from common household items like the ironing board to complex devices that have revolutionized manufacturing. The exercises that follow will help you identify just a few of these inventors and their inventions.

Match the inventors with their inventions by matching each system with its solution. (Not all the solutions will be used.)

1. Sara Boone	$x + y = 2$ $x - y = 10$ **E**	**A.** $(1, 4)$	automatic traffic signal
2. Sarah Goode	$x = 2 - y$ $2y + x = 9$ **D**	**B.** $(4, -2)$	eggbeater
3. Frederick M. Jones	$y = 2x + 6$ $y = -x - 3$ **G**	**C.** $(-2, 3)$	fire extinguisher
4. J. L. Love	$2x + 3y = 8$ $2x - y = -8$ **F**	**D.** $(-5, 7)$	folding cabinet bed
5. T. J. Marshall	$y - 3x = 9$ $2y + x = 4$ **C**	**E.** $(6, -4)$	ironing board
6. Jan Matzeliger	$y + 4 = 2x$ $6x - 3y = 12$ **J**	**F.** $(-2, 4)$	pencil sharpener
7. Garrett A. Morgan	$3x - 2y = -5$ $3y - 4x = 8$ **A**	**G.** $(-3, 0)$	portable X-ray machine
8. Norbert Rillieux	$3x - y = 12$ $y - 3x = 15$ **I**	**H.** $(2, -3)$	player piano
		I. no solution	evaporating pan for refining sugar
		J. infinitely many solutions	lasting (shaping) machine for manufacturing shoes

Answers

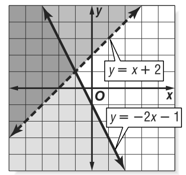
NAME ________________ DATE ________ PERIOD _____

7-5 Study Guide and Intervention

Graphing Systems of Inequalities

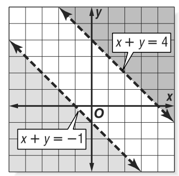
Systems of Inequalities The solution of a **system of inequalities** is the set of all ordered pairs that satisfy both inequalities. If you graph the inequalities in the same coordinate plane, the solution is the region where the graphs overlap.

Example 1 **Solve the system of inequalities by graphing.**
$y > x + 2$
$y \le -2x - 1$



The solution includes the ordered pairs in the intersection of the graphs. This region is shaded at the right. The graphs of $y = x + 2$ and $y = -2x - 1$ are boundaries of this region. The graph of $y = x + 2$ is dashed and is not included in the graph of $y > x + 2$.

Example 2 **Solve the system of inequalities by graphing.**
$x + y > 4$
$x + y < -1$

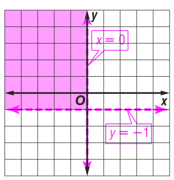


The graphs of $x + y = 4$ and $x + y = -1$ are parallel. Because the two regions have no points in common, the system of inequalities has no solution.

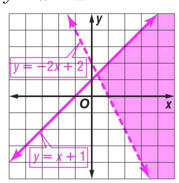
Exercises

Solve each system of inequalities by graphing.

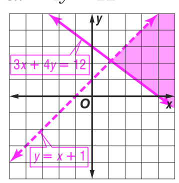
1. $y > -1$
$x < 0$



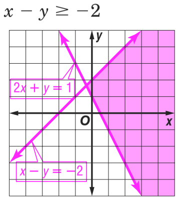
2. $y > -2x + 2$
$y \le x + 1$



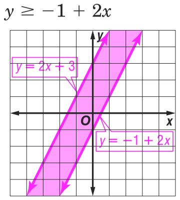
3. $y < x + 1$
$3x + 4y \ge 12$



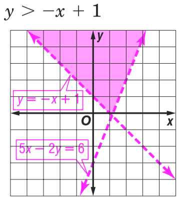
4. $2x + y \ge 1$
$x - y \ge -2$



5. $y \le 2x + 3$
$y \ge -1 + 2x$



6. $5x - 2y < 6$
$y > -x + 1$



Lesson 7-5

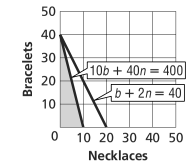
NAME ________________ DATE ________ PERIOD _____

7-5 Study Guide and Intervention *(continued)*

Graphing Systems of Inequalities

Real-World Problems In real-world problems, sometimes only whole numbers make sense for the solution, and often only positive values of x and y make sense.

Example **BUSINESS AAA Gem Company produces necklaces and bracelets. In a 40-hour week, the company has 400 gems to use. A necklace requires 40 gems and a bracelet requires 10 gems. It takes 2 hours to produce a necklace and a bracelet requires one hour. How many of each type can be produced in a week?**



Let n = the number of necklaces that will be produced and b = the number of bracelets that will be produced. Neither n or b can be a negative number, so the following system of inequalities represents the conditions of the problems.

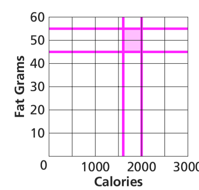
$n \ge 0$
$b \ge 0$
$b + 2n \le 40$
$10b + 40n \le 400$

The solution is the set ordered pairs in the intersection of the graphs. This region is shaded at the right. Only whole-number solutions, such as (5, 20) make sense in this problem.

Exercises

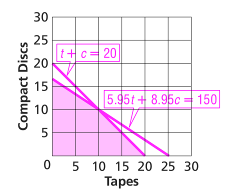
For each exercise, graph the solution set. List three possible solutions to the problem.

1. **HEALTH** Mr. Flowers is on a restricted diet that allows him to have between 1600 and 2000 Calories per day. His daily fat intake is restricted to between 45 and 55 grams. What daily Calorie and fat intakes are acceptable?



Sample answers: 1600 Calories, 45 fat grams; 1800 Calories, 50 fat grams; 2000 Calories, 55 fat grams

2. **RECREATION** Maria had \$150 in gift certificates to use at a record store. She bought fewer than 20 recordings. Each tape cost \$5.95 and each CD cost \$8.95. How many of each type of recording might she have bought?



Sample answers: 10 tapes, 9 CDs; 0 tapes, 16 CDs; 14 tapes, 5 CDs

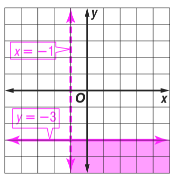
NAME ______________________ DATE __________ PERIOD _____

7-5 Skills Practice

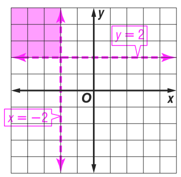
Graphing Systems of Inequalities

Solve each system of inequalities by graphing.

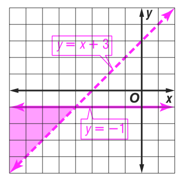
1. $x > -1$
$y \le -3$



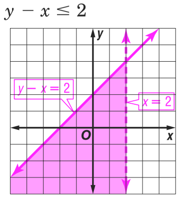
2. $y > 2$
$x < -2$



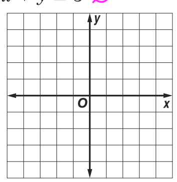
3. $y > x + 3$
$y \le -1$



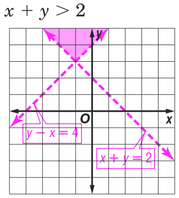
4. $x < 2$
$y - x \le 2$



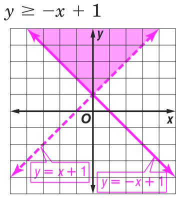
5. $x + y \le -1$
$x + y \ge 3$ ∅



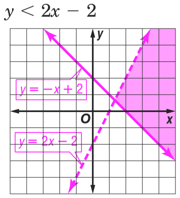
6. $y - x > 4$
$x + y > 2$



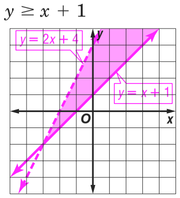
7. $y > x + 1$
$y \ge -x + 1$



8. $y \ge -x + 2$
$y < 2x - 2$

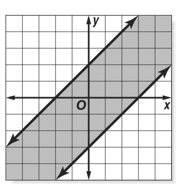


9. $y < 2x + 4$
$y \ge x + 1$



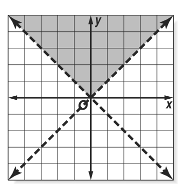
Write a system of inequalities for each graph.

10.



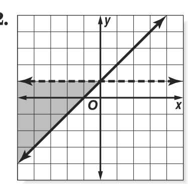
$y \le x + 2, y \ge x - 3$

11.



$y > -x, y > x$

12.



$y \ge x + 1, y < 1$

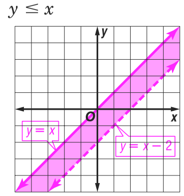
NAME ______________________ DATE __________ PERIOD _____

7-5 Practice (Average)

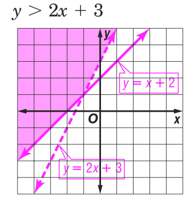
Graphing Systems of Inequalities

Solve each system of inequalities by graphing.

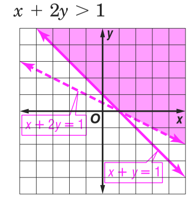
1. $y > x - 2$
$y \le x$



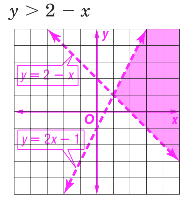
2. $y \ge x + 2$
$y > 2x + 3$



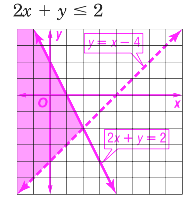
3. $x + y \ge 1$
$x + 2y > 1$



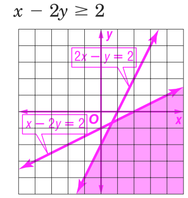
4. $y < 2x - 1$
$y > 2 - x$



5. $y > x - 4$
$2x + y \le 2$



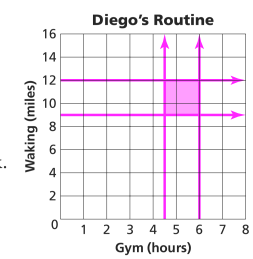
6. $2x - y \ge 2$
$x - 2y \ge 2$



FITNESS **For Exercises 7 and 8, use the following information.**

Diego started an exercise program in which each week he works out at the gym between 4.5 and 6 hours and walks between 9 and 12 miles.

7. Make a graph to show the number of hours Diego works out at the gym and the number of miles he walks per week.

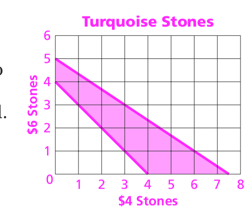


8. List three possible combinations of working out and walking that meet Diego's goals. **Sample answers: gym 5 h, walk 9 mi; gym 6 h, walk 10 mi, gym 5.5 h, walk 11 mi**

SOUVENIRS **For Exercises 9 and 10, use the following information.**

Emily wants to buy turquoise stones on her trip to New Mexico to give to at least 4 of her friends. The gift shop sells stones for either $4 or $6 per stone. Emily has no more than $30 to spend.

9. Make a graph showing the numbers of each price of stone Emily can purchase.



10. List three possible solutions. **Sample answer: one $4 stone and four $6 stones; three $4 stones and three $6 stones; five $4 stones and one $6 stone**

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7-5 Reading to Learn Mathematics

Graphing Systems of Inequalities

Pre-Activity **How can you use a system of inequalities to plan a sensible diet?**

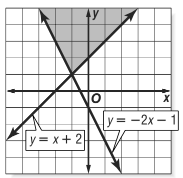
Read the introduction to Lesson 7-5 at the top of page 394 in your textbook.

The green section on the graph represents a range of **2000 to 2400** Calories a day and **60 to 75** grams of fat per day.

Reading the Lesson

Write the inequality symbols that you need to get a system whose graph looks like the one shown. Use <, ≤, >, or ≥.

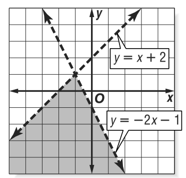
1.



y **≥** $x + 2$

y **≥** $-2x - 1$

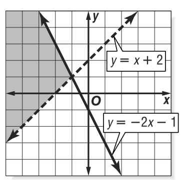
2.



y **<** $x + 2$

y **<** $-2x - 1$

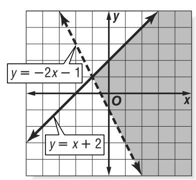
3.



y **>** $x + 2$

y **≤** $-2x - 1$

4.



y **≤** $x + 2$

y **>** $-2x - 1$

Helping You Remember

5. Describe how you would explain the process of using a graph to solve a system of inequalities to a friend who missed Lesson 7-5. **Graph each inequality on the same coordinate plane. The solutions are the ordered pairs for the points in both graphs.**

Lesson 7-5

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7-5 Enrichment

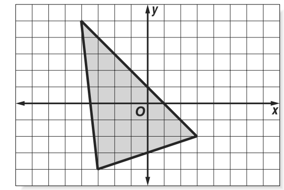
Describing Regions

The shaded region inside the triangle can be described with a system of three inequalities.

$y < 2x + 1$

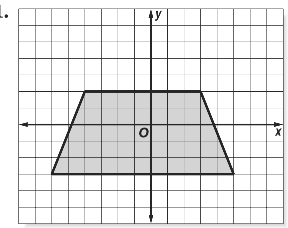
$y > \frac{1}{3}x - 3$

$y > 29x - 31$



Write systems of inequalities to describe each region. You may first need to divide a region into triangles or quadrilaterals.

1.

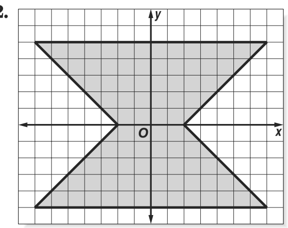


$y < \frac{5}{2}x + 12$

$y < -\frac{5}{2}x + \frac{19}{2}$

$y < 2$ $\quad y > -3$

2.

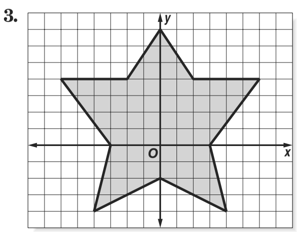


$y > -x - 2$ $\quad y < x + 2$

$y > x - 2$ $\quad y < -x + 2$

$y < 5$ $\quad y > -5$

3.



top: $y < \frac{3}{2}x + 7$, $y < -\frac{3}{2}x + 7$, $y > 4$

middle: $y < 4$, $y > 0$, $y > -\frac{4}{3}x - 4$, $y > \frac{4}{3}x - 4$

bottom left: $y < 4x + 12$, $y > \frac{1}{2}x - 2$, $y < 0$, $x < 0$

bottom right: $y < -4x + 12$, $y > -\frac{1}{2}x - 2$, $y < 0$, $x \not< 0$